Simulating the Detachment of Leading Edge Vortices on Drosophila Melanogaster using CFD

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1 Motivation

Initial CFD simulations of quasi-steady hovering flight of the Drosophila sp. have studied the effects of delayed rotation of the wings at the end of each stroke and it's effect on the resulting unsteady aerodynamic forces found throughout the wing [8]. Further studies were carried out using the same CFD model as before to determine the lift and power requirements for Drosophila Virilis [7]. To verify the validity of the simulation's results, the specific power that was calculated (the sum of aerodynamic and intertial power requirements normalized to the fruitfly muscle mass) was compared to the values retrieved by Dickenson on tethered fruitflies [1]. Although the periodic behavior of the 'translational' and rotational motions of the wing were heavily approximated, the comparison showed strong agreement. This lends support to the use of CFD models in the parametric design process used in the pioneering of microscale flapping wings [2]. It is of great importance to MEMs designers working on microscale flapping wing to study the detachment of Leading Edge Vortices (LEV). Solving the Navier-Stokes equations in the time-varying coordinates of the flapping wing proves a challenge computationally but offers the possibility of predicting the behavior of the LEV-shedding phenomenon to which flapping wings owe a majority of their lift. In addition to studying the shedding of LEVs, CFD simulations enable one to quickly assess the effect of wing shaping on vertical lift. Pre-

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liminary code was developed in MATLAB with these two potential studies in mind.

2 Governing Equations

The code that was developed to tackle the flapping wing problem solves the Navier-Stokes equations for a compressible fluid (written here in cartesian coordinate system):

$$\frac{\partial \hat{u}}{\partial t} = -\frac{\partial}{\partial x}(\hat{f} - \hat{f_v}) - \frac{\partial}{\partial y}(\hat{g} - \hat{g_v}) - \frac{\partial}{\partial z}(\hat{h} - \hat{h_v}) \tag{1}$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{2}$$

where $\hat{u}$, $\hat{f}$, $\hat{g}$, $\hat{h}$, $\hat{f_v}$, $\hat{g_v}$, $\hat{h_v}$

$$\hat{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{3}$$

$$\hat{f} = \begin{pmatrix} p + u^2 \\ uv \\ uw \end{pmatrix} \tag{4}$$

$$\hat{g} = \begin{pmatrix} vu \\ p + v^2 \\ vw \end{pmatrix} \tag{5}$$

$$\hat{h} = \begin{pmatrix} wu \\ wv \\ p + w^2 \end{pmatrix} \tag{6}$$

$$f_v = \nu \begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \\ \frac{\partial w}{\partial x} \end{pmatrix} \tag{7}$$

$$g_v = \nu \begin{pmatrix} \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial y} \end{pmatrix} \tag{8}$$

$$h_v = \nu \begin{pmatrix} \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial z} \end{pmatrix} \tag{9}$$

A constant density/incompressible fluid assumption can be applied to equation 2 - the PDE for the conservation of mass - which sends the $\frac{\partial \rho}{\partial t}$ term to zero. The resulting PDE can also be written as:

$$\nabla \bullet \hat{u} = 0 \tag{10}$$

This statement is now equivalent to saying that the mass flux entering a finite volume must equal the mass flux leaving that volume. The reynolds number for the flow about the flapping wings of Drosophila Melanogaster was determined to be 115 [3]. The low-reynolds number for the flow allows an additional assumption to be made: the flow is in the laminar regieme. This saves us from the added complexity of implementing a model for turbulent fluid flow. The following form of the Navier-Stokes equations is developed by Roger and Kwak [4]

$$[I_{tr}+(\frac{\partial \hat{R}}{\partial D})^{n+1,m}](D^{n+1,m+1}-D^{n+1,m}) = -\hat{R}^{n+1,m}-\frac{I_m}{\Delta t}(1.5D^{n+1,m}-2D^n+0.5D^{n-1}) \tag{11}$$

The derivative of the state vector with respect to time is discretized using a second-order three-point backward difference formula. The distinguishing feature of this equation is its implementation of artificial compressibilty.

$$\frac{\partial p}{\partial \tau} = -\beta \nabla \bullet \hat{u}^{n+1,m+1} \tag{12}$$

This equation, although physically meaningless with the inclusion of the derivative of pressure on the left hand side, enforces the desired incompressibility assumption which lead us to equation 10. This is achieved through carrying the form of the Navier-Stokes equations forward in artificial time. This requires adding a pseudotime derivative (a derivative with respect to artificial time) to all four of the Navier-Stokes equations. The three other equations that complement equation 12 can be written in a similar fashion to the one reproduced below (in its discretized form).

$$\frac{1.5\hat{u}^{n+1,m+1} - 1.5\hat{u}^{n+1,m}}{\Delta t} = \frac{-\hat{r}^{n+1,m+1} - 1.5\hat{u}^{n+1,m} - 2\hat{u}^n + 0.5\hat{u}^{n-1}}{\Delta t} \tag{13}$$

Together, equation 12 and the three equations that take on the form of 13 can be written compactly in the delta form (equation 1) mentioned earlier:

where D, R, E, F, G, E_v, F_v, G_v are

$$D = \begin{pmatrix} p \\ u \\ v \\ w \end{pmatrix} \tag{14}$$

$$\hat{R} = \frac{\partial}{\partial x}(F - F_v) + \frac{\partial}{\partial y}(G - G_v) + \frac{\partial}{\partial z}(H - H_v); \tag{15}$$

$$F = \begin{pmatrix} \beta u \\ p + u^2 \\ uv \\ uw \end{pmatrix} \tag{16}$$

$$G = \begin{pmatrix} \beta v \\ vu \\ p + v^2 \\ vw \end{pmatrix} \tag{17}$$

$$H = \begin{pmatrix} \beta w \\ wu \\ wv \\ p + w^2 \end{pmatrix} \tag{18}$$

$$F_v = \nu \begin{pmatrix} 0 \\ \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \\ \frac{\partial w}{\partial x} \end{pmatrix} \tag{19}$$

$$G_v = \nu \begin{pmatrix} 0 \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial y} \end{pmatrix} \tag{20}$$

$$H_v = \nu \begin{pmatrix} 0 \\ \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial z} \end{pmatrix} \tag{21}$$

and where dR/dD consists of the jacobians of the convective and viscous flux terms. The jacobian arises from a common technique used in solving the Navier-Stokes equations numerically. This technique linearizes the residuals of each equation so that one can write the equations in implicit form and from this the systems of linear equations can be solved numerically using conventional methods.

3 Numerical Method

Equation 11 is written in a form that can be solved using an procedure called the Gauss-Seidel line-iteration. The viscous fluxes are discretized with a second order central difference scheme while the convective fluxes are discretized using a cell-centered scheme that treats the fluxes across each node (each finite volume) as conservative variables. This discretization proves much more stable as it provides numerical dissipation automatically. This discretization is formed as follows for the flux passing through the faces corresponding to x-ward normal vectors:

$$\frac{[\hat{f}_{i+1/2} - \hat{f}_{i-1/2}]}{\Delta x} = 0 \tag{22}$$

$$\hat{f}_{i+1/2} = \frac{1}{2}[f(q_{i+1}) + f(q_i)] - \frac{1}{2}[\Delta f^{+}_{i+1/2} - \Delta f^{-}_{j+1/2}] \tag{23}$$

$$\Delta f^{\pm}_{i+1/2} = A^{\pm}(\hat{q})\Delta q_{i+1/2} \tag{24}$$

the last equation 24, is the flux difference term found in equation 23 and it can be calculated provided that one can find a matrix A that is the jacobian of the flux vector with respect to the state vector D. This jacobian must also carry with it other properties all captured by the so called 'property U'. Creating a jacobian matrix with 'property U' allows the jacobian to be representative of the change in flux with respect to state vectors at the interface between the two nodes it is responsible for. P. L. Roe developed a method to produce such jacobians that is implemented on the Euler equations [3] but can be partly extended toward the navier-stokes equations and its convective fluxes.

4 Results and Further Work

The main problem facing the code now in its development is that upon attempting to solve the linear system of equations according to the Gauss-Seidel line-iteration, the matrix cannot be numerically inverted because it is found by linsolve() in MATLAB to be singular. This prevents results from being produced for the simple 3D channel problem.

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